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A task-based type synthesis of novel 2T2R parallel mechanisms

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ABSTRACT

This article discusses a generic synthesis methodology using the tools of screw theory and the concept of motion pattern, application of which is illustrated by the synthesis of novel parallel mechanisms suited to the task of needle manipulation derived from a medical application. From the minimal 2T2R mobility required for such task, two different motion patterns are derived and have permitted the synthesis of two novel 2T2R parallel mechanisms. Application of the kinematic inversion principle is demonstrated for special cases where the motion pattern is defined with respect to the moving platform. Throughout the synthesis process, a set of preference rules qualifying the architecture kinematic complexity is utilized in order to obtain parallel mechanisms with reduced complexity. These rules are related to the geometric conditions on the placement of the joint axes within and between the synthesized legs. Each parallel mechanism candidate is tested for conditions of full-cycle mobility. Finally, the selection of actuated joints is validated by analyzing the full direct Jacobian matrix.

1. Introduction

Generally, parallel mechanisms are synthesized beforehand and the applications to which they might be suited, are discovered later. An approach based on the application motivated task-based synthesis has the potential to synthesize novel parallel mechanisms which are better suited to the needs of the application.

There are several methods available for the synthesis of parallel mechanisms which may fall into two large categories. The first one is based on the study of the motion of the moving platform, resulting from the intersection of motions provided by each leg. Depending on the representation chosen to describe the motion, powerful synthesis methods have been proposed based on the group theory [1–4] or based on the theory of linear transformations [5,6]. The second category uses a dual approach, since the focus is rather set on the constraints applied to the moving platform, which are formed by the union of the constraints imposed by each leg. Screw theory is often used for this approach since it can describe in a unified manner both motions and constraints acting on the rigid bodies. Several reference works have recently appeared on the subject of the type synthesis of the parallel mechanisms based on the screw theory [7–9]. The synthesis of mechanisms using screw theory is based on the principle of reciprocity which implies finding the reciprocity conditions between screw systems, namely wrench and twist systems. This approach can be conducted algebraically [10] and involves calculating the null bases of a given set of wrenches to derive the reciprocal twist systems. However, there also exists more geometric deductive reasoning leading to similar results which are widely used in research works focused on type synthesis of parallel mechanisms [7,11].

The synthesis of mechanisms often starts with a mobility requirement pertaining to a given task. However, the concept of mobility [12], stated as the number of independent coordinates to define the configuration of a mechanism, is not sufficient to characterize fully and accurately the motion capability of the mechanism. A more comprehensive description can be gained by

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considering the DOF partitioning into their rotational and translational components [13,14] The above approach is quite generic. It is also applicable to mechanisms with variable mobility over the workspace and in singular configurations. Still, these methods are applied at instantaneous configurations and are not feasible to apply for each point of the workspace. In this paper, only mechanisms with invariable mobility are considered and this eliminates the need for calculating instantaneous mobilities over the whole workspace. Since screw theory produces mechanisms with instantaneous mobility only, some criterion has to be utilized to ascertain the invariability of the mobility. In this context, the concept of full-cycle mobility [5,8] is used as a criterion and can be verified using geometric properties via the screw theory instead of algebraic or numeric calculations.

Even with the above mobility partitioning and the validation of the full-cycle mobility criterion, there could still exist an ambiguity in its description. This is specially the case in the context of the mechanism synthesis presented in this paper, where two different motion patterns are identified corresponding to the same DOF partitioning. This problem of mobility specification can be disambiguated using the tools of screw theory which provide a deeper insight into the motion of the end-effector and the constraints applied to it. The concept of motion pattern along with the virtual chain approach introduced by Kong and Gosselin [11] helps to solve this issue of mobility specification. Though the concept of motion pattern is utilized in this paper, the synthesis method using virtual chains is not employed since it is not always possible to find a virtual chain for every motion pattern.

The synthesis of parallel mechanisms is not complete until the choice of the actuated joints has been validated. For satisfying the constraints as well as the mobility of the parallel mechanism, a minimum number of legs is required. Exceeding this number of legs imposes additional constraints on the mechanism but often it becomes necessary for locating the actuated joints near the base and enhancing the rigidity of the mechanism. These overconstraints introduce additional geometrical conditions on the placement of the joints axes in the mechanism. The effect of having these geometric conditions not met need to be investigated for ascertaining the mobility of the mechanism. As pointed out in [15], the construction of certain parallel mechanisms turns out to be a challenging task because the required geometric conditions are very difficult to meet and consequently the fabricated mechanism may not exhibit the expected mobility. Therefore, a set of preference rules will be proposed based on the feasibility and practical considerations, in order to obtain parallel mechanism candidates with reduced geometrical complexity.

This paper is divided into three main sections. The next section introduces the steps of a generic synthesis methodology proposed for conducting a task-based mechanism synthesis. In Section 3, the task of needle manipulation motivated by a medical application is described and the required minimal 2T2R mobility is discussed. Utilizing the task of needle manipulation, the task-based synthesis methodology is illustrated step by step in detail. The concept of motion patterns is shown to disambiguate the definition of the mobility, which led to the derivation of the two novel motion patterns corresponding to the same mobility. The possibility of motion pattern definition with respect to a moving reference frame is evoked and a solution is offered by means of the kinematic inversion principle. In Section 4, the type synthesis reveals novel 2T2R parallel mechanisms which are not reported from previous papers on the synthesis of 2T2R parallel mechanisms.

2. Generic task-based synthesis methodology

Since the objective of a task-based synthesis procedure is to obtain mechanisms in the context of a practical application, one does not need to enumerate all possible candidates but to find the best candidate with the least architectural complexity.

2.1. Preference rules for leg composition

During the synthesis process, the following *preference rules* will be followed for the selection of suitable legs:

1. Types of joints: The generated legs should avoid prismatic joints if possible.
2. Redundancy: The generated legs will be non-redundant to avoid extra actuators.
3. Overconstraints: The number of overconstraints for the parallel mechanism should be minimized.
4. Geometric conditions within a leg (decreasing order of preference):
   (a) Two revolute joints parallel to each other ($(RR)_p$).
   (b) Three revolute joints parallel to each other ($(RRR)_p$).
   (c) Two revolute joints intersecting each other at one point ($(RR)_i$).
   (d) Three revolute joints intersecting each other at one point ($(RRR)_i$).
5. Geometric conditions between legs (decreasing order of preference):
   (a) Parallelism between sets of $(RR)_p$.
   (b) Parallelism between sets of $(RRR)_p$.
   (c) Sets of $(RR)_i$ intersect at one common point.
   (d) Sets of $(RRR)_i$ intersect at one common point.

2.2. Synthesis procedure

Though type synthesis with the screw theory is a well established method, it has to be adapted for task specific synthesis problems.

Step 1: Identification of the mobility requirements from the task definition
   (a) Identification of all possible motion patterns

Step 2: Corresponding to each motion pattern, identification and decomposition of its wrench system
Step 3: Generation of legs, serial or closed loops, conforming to the possible combination of the wrench systems
(a) Identification of the legs with least architecture complexity based on the rules in Section 2.1
Step 4: Generation of candidate parallel mechanisms by possible combination of legs
(a) Check for the full-cycle mobility criterion
(b) Check for the validity of the choice of actuated joints
Step 5: If steps 4a and 4b are not satisfied go to step 3.

Though the steps 3–4 are common in the synthesis process using screw theory, a task-based synthesis process requires steps 1 and 2 before proceeding with the next steps.

3. Application: task-based synthesis procedure

3.1. Medical application: needle manipulation — task definition

Interventional radiology is a medical specialty where the use of surgical needles is very common. One of the key gestures of the radiologist is the needle manipulation in free space. Needle manipulation can be seen as the placement of a line $\mathcal{L}$, supporting the needle axis in 3D space, using the mechanism to be synthesized. Let $\mathcal{F}_b = (O_b, \mathbf{x_b}, \mathbf{y_b}, \mathbf{z_b})$be a fixed reference frame and $\mathcal{F}_f = (O_f, \mathbf{x_f}, \mathbf{y_f}, \mathbf{z_f})$ be the frame affixed to the moving platform of the mechanism. In this specific context, two practical points can serve to define $\mathcal{L}$, namely an entry point E on the patient's skin and the point O_f attached to the moving platform such that the direction of $\mathcal{L}$ coincides with $\mathbf{z_f}$. The entry point E is specified by the radiologist from the images acquired in a pre-operative stage, but it is sometimes necessary to readjust the position of E. Assuming that the point O_b is the initial targeted entry point, a zone of operation around O_b is defined by a plane Π tangent to the skin at point O_b with a normal vector $\mathbf{z_b}$, as depicted in Fig. 1. After fixing the point E in Π, the needle is rotated around this point to obtain the required orientation for reaching the target organ.

Therefore, the kinematic structure of the mechanism for such a task has at least four DOFs or more precisely the mechanism may have a 2T2R mobility. At the start of the procedure, the radiologist may wish to tilt the needle axis around the entry point while the needle has just been slightly inserted. Then, a key requirement is the presence of a remote center of motion (RCM) point which coincides with the point of entry on the patient's skin. This facilitates the orientation of the mechanism without causing any tissue lacerations.

A number of dedicated robotic systems have been developed during the last decade but there is still a need for improvements in the compactness and the functionality integration. While the table-mounted systems [16,17] usually satisfy all the requirements of the needle manipulation, they require additional passive or active gross pre-positioning units, which makes the overall system less compact and bulky. The patient-mounted systems [18–20] either lack certain DOFs or they have redundant DOFs, with the exception of the new LPR [21] system which has a minimal 2T2R mobility. Not every such system is designed to ensure the entry point on the patient's skin as the RCM point. It is expected that the benefits of a well-chosen parallel mechanisms could help to reduce the size, the weight, the number of actuators and the overall complexity of such medical robotic assistants. Few parallel mechanisms with 2T2R mobility have been reported in the literature. For example, the mechanisms presented in [22–25] are developed for applications other than the needle manipulation and hence, their motion pattern does not satisfy the

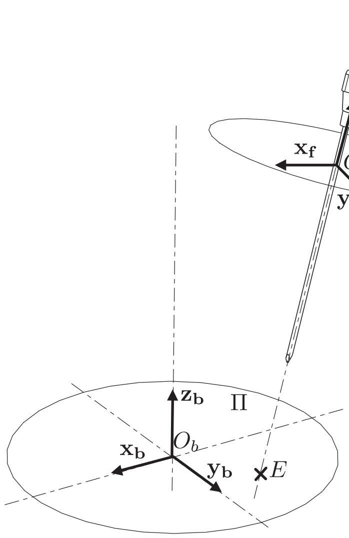


Fig. 1. Task definition.

requirements of the targeted procedure. The synthesis presented in this paper will ensure that the proposed mechanisms have a RCM point.

3.2. Identification of the required wrench system/motion pattern

As a first step, the wrench system based on the motion pattern needs to be identified. Twists and wrenches of pitch h are denoted as $\$^h$ and $\hat{\$}^h$ respectively. Accordingly, wrench systems of order n formed by zero and infinite pitch wrenches are denoted as n–$\hat{\$}^0$ and n–$\hat{\$}^\infty$. A wrench generator corresponding to the desired motion pattern will be referred to as $\hat{\$}^\infty_{mp}$.

Based on the 2T2R mobility described before, the required twist system for the mechanism is derived to be 2–$\hat{\$}^0$–2–$\hat{\$}^\infty$. Hence, the reciprocal wrench system of the mechanism turns out to be 1–$\hat{\$}^0$–1–$\hat{\$}^\infty$ and is generated by two wrenches: i) a zero-pitch wrench $\hat{\$}^0_{mp}$ whose axis coincides with the line (E,z_b) and ii) an infinite-pitch wrench $\hat{\$}^\infty_{mp}$. These two constraint wrenches describe the required motion pattern for the task.

The direction of $\hat{\$}^\infty_{mp}$ need not remain fixed and it can vary with the platform configuration. Two cases can be identified, as shown in Fig. 2(a) and (b), on the basis of two different motion patterns mp_1 and mp_2 respectively. In Case-1, the wrench $\hat{\$}^\infty_{mp_1}$ is configuration dependent but neither fixed relative to the base nor to the platform. In Case-2, the wrench $\hat{\$}^\infty_{mp_2}$ is also configuration dependent but has a direction always parallel to $\mathbf{z_f}$. In this Case-2, the mechanism prohibits rotation around the needle's axis $\mathbf{z_f}$ in all configurations.

As it can be noticed from the above results, two cases have the same mobility with the same DOF partitioning. But still, a clear distinction can be arrived between the two cases by using the concept of motion patterns. As it will be shown in later sections, different mechanism architectures will be synthesized from these two cases.

3.3. Decomposition of the wrench system

The generation of the wrench system 1–$\hat{\$}^0$–1–$\hat{\$}^\infty$ may be either done directly or using the combinations of elementary wrench systems 1–$\hat{\$}^0$ and 1–$\hat{\$}^\infty$. In the following discussion, the reciprocity of twist and wrench systems is used to synthesize legs providing a desired wrench system taking into account the aforementioned *preference rules* in Section 2.1.

Legs with the expected properties will be composed of serial kinematic subchains, which include only revolute and prismatic joints. The principle of reciprocity imposes the following geometric conditions on the placement of the revolute and the prismatic joints within the subchains depending on the wrench system to be generated:

Condition-1 Each prismatic joint needs to be perpendicular to the direction of $\hat{\$}^0_{mp}$.
Condition-2 Each revolute joint axis should be coplanar with the axis of $\hat{\$}^0_{mp}$. This condition can be further refined into (a) Each revolute joint should either intersect the axis of the $\hat{\$}^0_{mp}$ or/and (b) Each revolute joint should be parallel to the axis of $\hat{\$}^0_{mp}$.
Condition-3 Each revolute joint should be perpendicular to the direction of the $\hat{\$}^0_{mp}$.

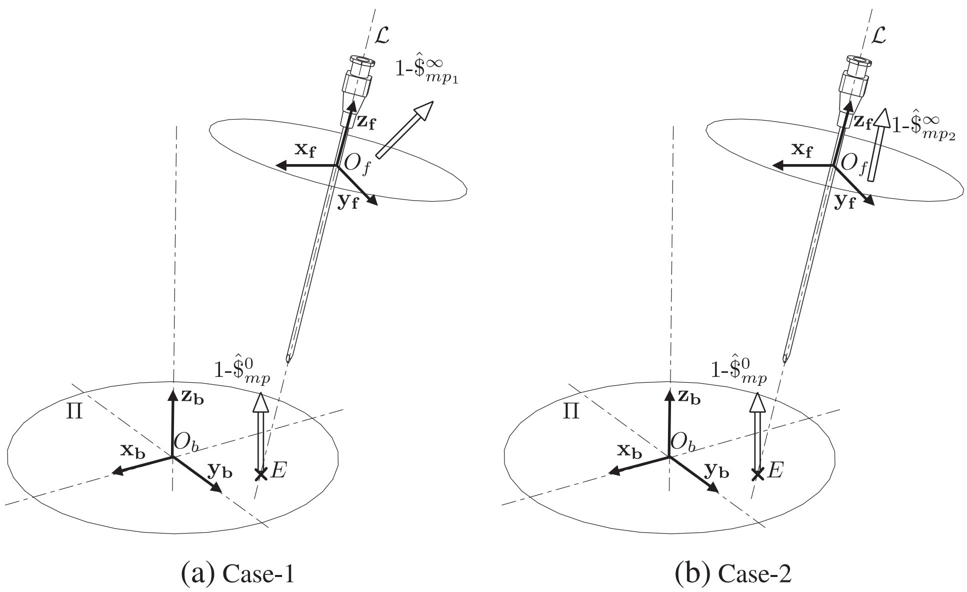


Fig. 2. Identified wrench systems.

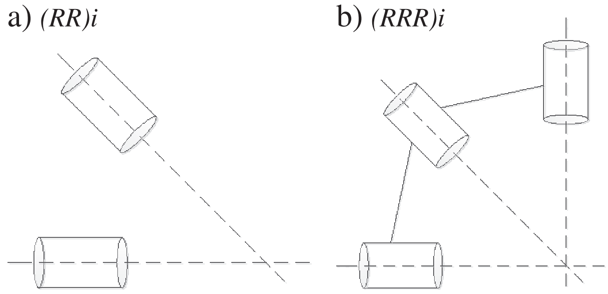


Fig. 3. Subchains with intersecting axes.

As can be noticed, Condition-1 applies only for prismatic joints whereas Conditions-2 and 3 apply for the revolute joints. From Condition-1, there can be one or at most two prismatic joints which are perpendicular to the axis of $\hat{\$}^0_{mp}$. From Condition-2(a), two subchains as shown in Fig. 3(a) and (b), can be identified where either two or three revolute joints are intersecting the axis of $\hat{\$}^0_{mp}$. From Condition-2(b), two groups as shown in Fig. 4(a) and (b) can be identified, where either two or three revolute joints are parallel to the axis of $\hat{\$}^0_{mp}$. In Figs. 3(b) and 4(b), the three revolute axes are not coplanar.

3.4. Synthesis of legs corresponding to wrench system $1-\hat{\$}^0-1-\hat{\$}^\infty$

For a non-redundant serial chain with wrench system $1-\hat{\$}^0-1-\hat{\$}^\infty$, the order of the wrench system is two. Hence, the number of joints amounts to $n_{\text{joints}} = 6 - 2 = 4$.

Unfortunately, there does not exist a serial chain with four revolute joints, which satisfies simultaneously the Conditions-2 and 3. The only serial chain with wrench system, $1-\hat{\$}^0-1-\hat{\$}^\infty$ corresponding to Fig. 2(a) has two prismatic joints conforming to Condition-1, attached to the base and a $(RR)_i$ subchain attached to the platform, as shown in Fig. 5. There does not exist a serial chain with wrench system $1-\hat{\$}^0-1-\hat{\$}^\infty$ corresponding to Fig. 2(b).

Exhaustive enumeration of serial chains with wrench system $1-\hat{\$}^0$ and $1-\hat{\$}^\infty$ is not done here as it can be readily found in literature by several authors [8,7,26]. In the following two sections, serial chains with only revolute joints are considered.

3.5. Synthesis of legs corresponding to wrench system $1-\hat{\$}^0$

Any non-redundant serial chain with this wrench system must have five joints. For a leg with only revolute joints which must satisfy Condition-2, there exist two possibilities, by combining the subchains listed in Figs. 3 and 4:

1. $(RRR)_p(RR)_i$
2. $(RR)_p(RRR)_i$.

The above mentioned combinations need to satisfy the requirement of the wrench system as well as that of the required mobility. Serial chain number 1 was selected, since it has the simplest geometric conditions on the placement of its joints based on the *preference rules* set in Section 2.1. The resulting serial leg is formed by a $(RRR)_p(RR)_i$ chain.

At this step, it needs to be verified that the selected serial leg also has the correct twist system or the required mobility. When the twist systems $1–\$^0–2–\$^\infty$ and $2–\$^0$ for the two subchains are combined, as shown in Fig. 6, the desired $3–\$^0–2–\$^\infty$ twist system for the serial chain is obtained, provided that the adjacent axes of the two subchains are set skewed. Additionally, the order of composition of these revolute joints in the subchains is important for the correct motion pattern and the condition of full-cycle

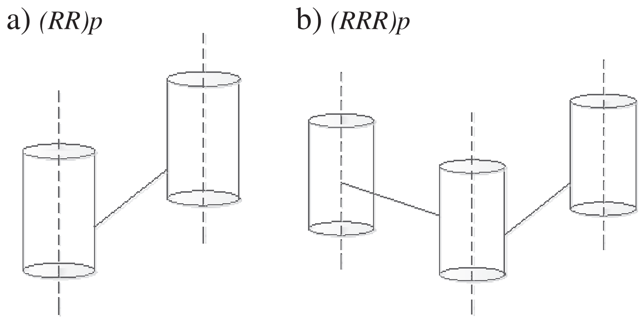


Fig. 4. Subchains with parallel axes.

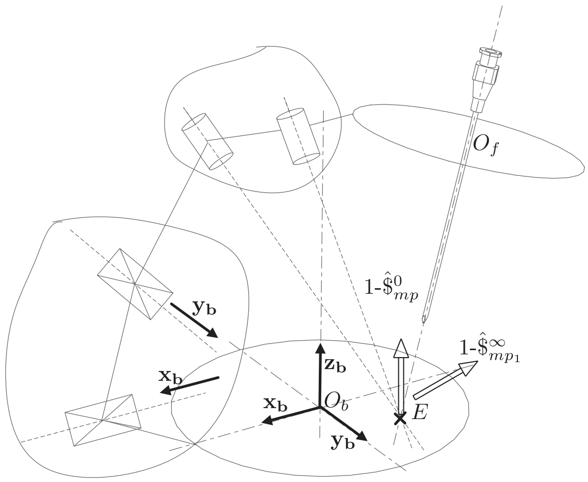


Fig. 5. Selected 2P2R chain with wrench system $1-\hat{\$}^0-1=\hat{\$}^\infty$.

mobility. Hence, the subchains $(RRR)_p$ and $(RR)_i$ need to be attached to the base and to the platform respectively. These subchains respectively ensure that Condition-2(b) and Condition-2(a) are met. The resulting chain prohibits one translational DOF along z_b and allows for rotations in all three directions.

3.6. Synthesis of legs corresponding to wrench system $1-\hat{\$}^\infty$

The synthesis of legs with wrench system $1-\hat{\$}^\infty$ is done to comply with the desired wrench systems $\hat{\$}^\infty_{mp_1}$ and $\hat{\$}^\infty_{mp_2}$, as depicted in Fig. 2(a) and (b).

Case 1. Wrench $\hat{\$}^\infty_{mp_1}$

A serial chain with this wrench system has five joints. From Condition-3, it is known that all the revolute joints need to be perpendicular to the direction of $\hat{\$}^\infty_{mp_1}$. There exists only one serial chain with only revolute joints, which satisfies this geometric condition. It consists of two subchains $(RRR)_p$ and $(RR)_p$. Again, the arbitrary permutation of individual revolute joints within the leg is not allowed and only the permutation of the two subchains is permitted. Hence, we can have either $(RRR)_p$ $(RR)_p$ or $(RR)_p$ $(RRR)_p$ as possible placements of revolute joints from the base to platform. The former arrangement is shown in Fig. 7.

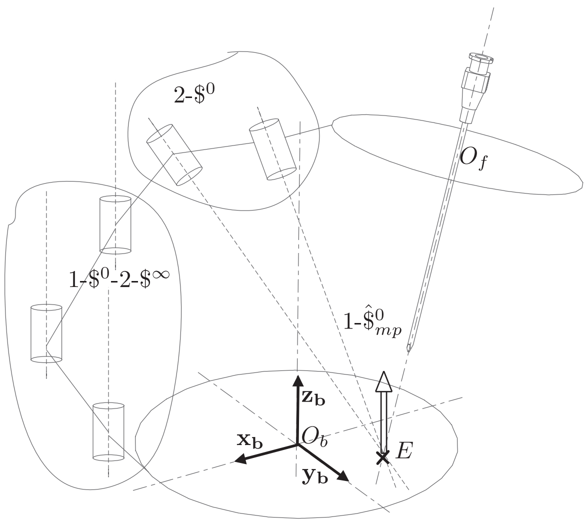


Fig. 6. Selected $(RRR)_p(RR)_i$ chain with wrench system $1-\hat{\$}^0$.

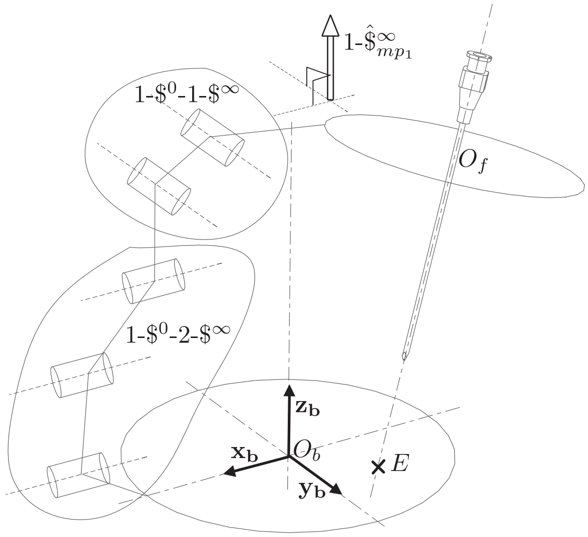


Fig. 7. Selected chain with wrench system $1-\hat{\$}^{\infty}_{mp_1}$.

When the twist systems 1–$\$^0$–2–$\$^\infty$ and 1–$\$^0$–1–$\$^\infty$ for the two subchains are combined, the desired 2–$\$^0$–3–$\$^\infty$ twist system for the serial chain is obtained, provided that the adjacent axes of the two subchains are set skewed. The resulting chain prohibits one DOF of rotation and allows for translations in three directions.

Case-2. Wrench $\hat{\$}^{\infty}_{mp_2}$

Generally, mobility specifications are formulated with respect to the fixed reference frame $\mathcal{F}_b$ rather than a moving reference frame. But in Case-2, as shown in Fig. 2(b), the direction of $\hat{\$}^{\infty}_{mp_2}$ is coincident with vector $\mathbf{z_f}$ which is associated with the frame $\mathcal{F}_f$ attached to the platform. One can note that the type synthesis of parallel mechanisms implicitly assumes that the constraints or

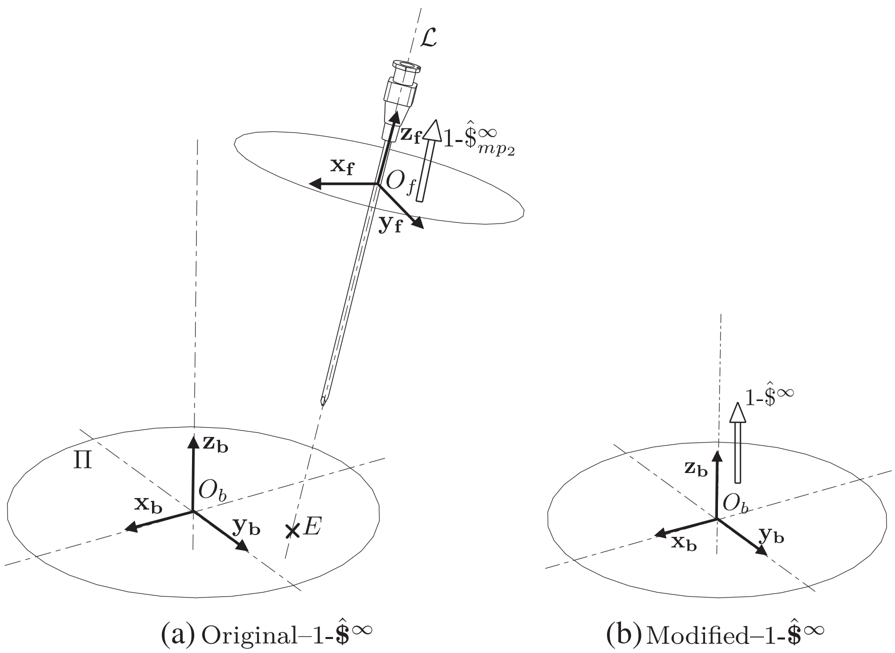


Fig. 8. Modification of the wrench system $1-\hat{\$}^{\infty}$.

motion patterns are defined with respect to a fixed reference frame. For taking into account the fact that the wrench system needs to be defined with respect to the platform frame $\mathcal{F}_f$, the kinematic inversion principle can be utilized during the synthesis process. After performing leg synthesis with respect to a fixed reference $\mathcal{F}_b$, the order of the joints within the leg is reversed from the base to the platform. The resulting leg obtained through kinematic inversion then features the desired wrench system expressed with respect to $\mathcal{F}_f$.

Hence, one has to start with the synthesis of legs with the wrench system $1-\hat{\$}^{\infty}$ along the direction of $\mathbf{z_b}$ as shown in Fig. 8(b).

Considering only revolute and prismatic joints, there exists no open loop serial chain with five joints, which corresponds to the wrench system $1-\hat{\$}^{\infty}$ with a fixed direction relative to $\mathcal{F}_b$. The legs obtained in Case-1 have the wrench system $1-\hat{\$}^{\infty}_{mp_1}$ but their direction is fixed neither relative to $\mathcal{F}_b$ nor $\mathcal{F}_f$.

For the synthesis of this leg, the synthesis of serial chains with the wrench system $1-\hat{\$}^{0}-1-\hat{\$}^{\infty}$ is first described. Such a four DOF leg has the desired component $1-\hat{\$}^{\infty}$ as part of its wrench system. The wrench system $1-\hat{\$}^{0}-1-\hat{\$}^{\infty}$ is equivalent to the linear combination of two wrench systems $1-\hat{\$}^{0}_1$ and $1-\hat{\$}^{0}_2$ with parallel directions, as shown in Fig. 10. The direction of the resulting $1-\hat{\$}^{\infty}$ wrench system is then perpendicular to the plane formed by the axes of $1-\hat{\$}^{0}_1$ and $1-\hat{\$}^{0}_2$. The synthesis of the serial chain with wrench system $1-\hat{\$}^{0}$ was earlier done in Section 3.5, with the only difference that the axis of $1-\hat{\$}^{0}$ is now parallel to the plane $(Ob,\mathbf{x_b},\mathbf{y_b})$, as demonstrated in Fig. 9(a).

Thus, a kinematic loop $B_1B_2A_2A_1$ generating a wrench system $1-\hat{\$}^{0}-1-\hat{\$}^{\infty}$ is obtained, as shown in Fig. 10(a). Finally, the objective is to obtain a leg with a wrench system $1-\hat{\$}^{\infty}$, as shown in Fig. 8(b). In the kinematic loop $B_1B_2A_2A_1$, to remove the constraint wrench $1-\hat{\$}^{0}$, it suffices to add a prismatic joint to the line B_1B_2, with a direction parallel to the axes of $1-\hat{\$}^{0}_1$ and $1-\hat{\$}^{0}_2$. Thus, after the addition of one prismatic joint to the kinematic loop $B_1B_2A_2A_1$, only one constraint wrench remains (that of $1-\hat{\$}^{\infty}$) with a direction parallel to $\mathrm{z_b}$ and defined with respect to $\mathcal{F}_b$.

Thus, a closed loop kinematic chain $2-(RR)_i(RRR)_p-P$ with a wrench system $1-\hat{\$}^{\infty}_{mp_2}$ is obtained, as shown in Fig. 10(b). The wrench system $1-\hat{\$}^{\infty}_{mp_2}$ has a direction parallel to $\mathbf{z_f}$ and is defined with respect to $\mathcal{F}_f$, as required. To obtain another possibility for the closed loop kinematic chain, it suffices to replace the $(RR)_i(RRR)_p$ chain with $(RRR)_i(RR)_p$, which imposes more stringent geometrical relationships than the former.

4. Synthesis of novel 2T2R mechanisms

In the previous Sections 3.4 to 3.6, the legs with reduced complexity were obtained for a given constraint wrench taking into account the *preference rules* set in Section 2.1. In this section, several candidates are now successively examined with respect to the geometric conditions between legs, the condition of full-cycle mobility and the validity of the selection of the actuated joints.

4.1. Architecture candidate I

For a fully parallel and symmetrical mechanism with identical legs, this 2T2R parallel mechanism would consist of four 2P2R legs with wrench system $1-\hat{\$}^{0}-1-\hat{\$}^{\infty}$, as depicted in Fig. 5. In this case, the 4-legged 2P2R parallel mechanism has overconstraints owing to the additional three legs.

4.1.1. Geometric conditions between and within the legs

Without loss of generality, within the leg j with $j = 1\ldots4$, one can assume that prismatic joints are parallel to a plane Π. The axes of the two revolute joints of the subchain $(RR)_i$ of leg j intersect each other in E and form the plane Π_j.

Between the legs, a group of prismatic joints are parallel to plane Π. This condition ensures that the axes of $\hat{\$}^{0}_{mp}$ are parallel for all legs. The revolute joints of the subchain $(RR)_i$ in each leg intersect in E, which is common to all legs. This condition ensures that

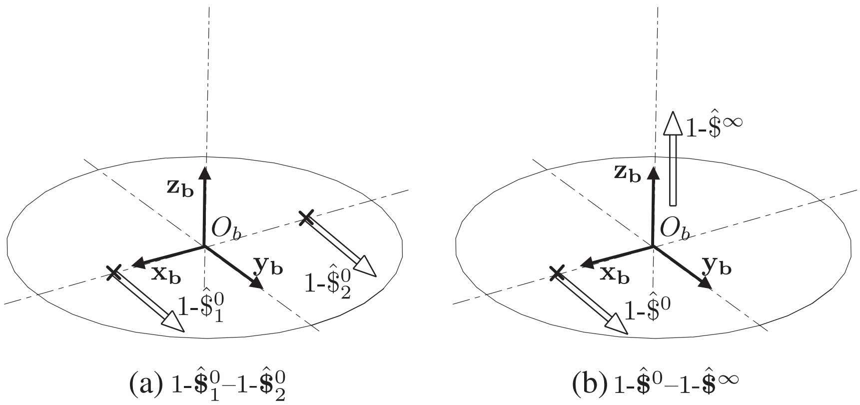
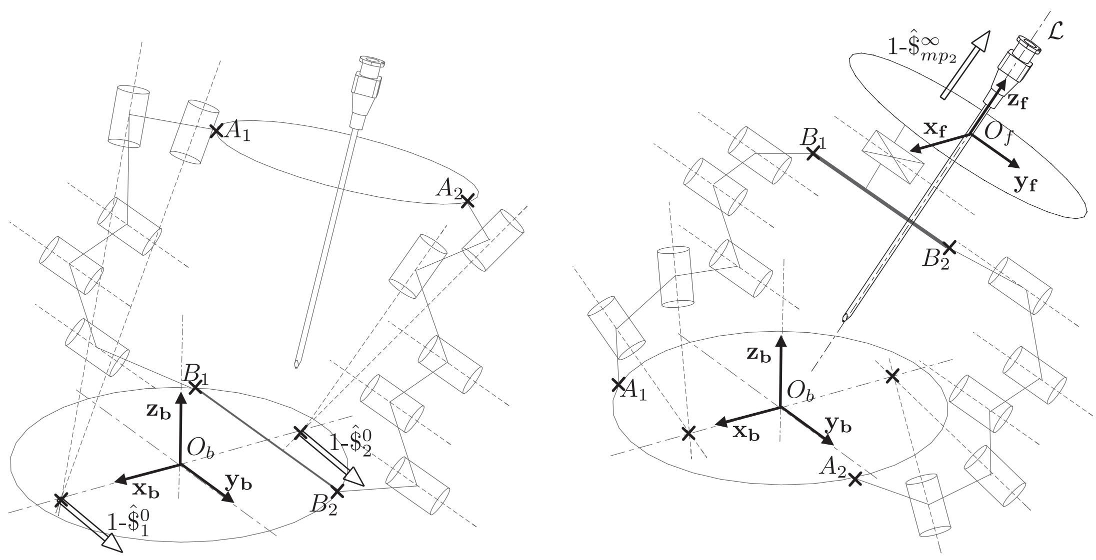


Fig. 9. Two equivalent wrench systems $1-\hat{\$}^{0}_1-\hat{\$}^{0}_2$ and $1-\hat{\$}^{0}-1-\hat{\$}^{\infty}$.



(a) closed kinematic chain (b) Inverted closed kinematic chain

Fig. 10. Kinematic inversion.

each axis of $\hat{\$}^{0}_{mp}$ has a common point of origin. To make certain that each leg generates the same $\hat{\$}^{\infty}_{mp_1}$ wrench system, the planes Π_j need to be parallel in every configuration. As every plane Π_j pass through the point of entry E, they have to be coincident.

4.1.2. Mobility check

A parallel mechanism synthesis conducted using screw theory does not necessarily produce mechanisms with full-cycle mobility, hence this has to be investigated. If all the conditions within and between legs, listed in Section 4.1.1 are satisfied in all the configurations, then the parallel mechanism is said to have full-cycle mobility. Unfortunately, the last condition which requires that the planes Π_j be coincident can only be satisfied in one configuration. Hence this 4-2P2R mechanism does not have full-cycle mobility and is not a valid candidate. As the 2P2R chain is the only possible leg choice with the $1-\hat{\$}^{0}-1-\hat{\$}^{\infty}$ wrench system, a fully parallel mechanism with identical legs cannot be obtained.

4.2. Architecture candidate II

As it could be concluded that the synthesis of 2T2R parallel mechanisms with identical legs is not possible, legs derived from elementary wrench systems are used to synthesize the 2T2R parallel mechanisms with different legs. In the above scenario, as a compromise between the rigidity and workspace, the number of overconstraints n_{oc} is chosen to be equal to one.

For a non-overconstrained mechanism, there would be only two legs, each corresponding to the elementary wrench systems $1-\hat{\$}^{0}$ and $1-\hat{\$}^{\infty}$. With one overconstraint, the number of legs rises to three and the supplementary leg has to be chosen with a $1-\hat{\$}^{0}$ or $1-\hat{\$}^{\infty}$ wrench system.

The wrench system of this second candidate corresponds to the one depicted in Fig. 2(a). As can be observed from Fig. 11, the chain A_1B_1 is a $(RRR)_p(RR)_i$ leg which has the wrench system $1-\hat{\$}^{0}$. The chains A_2B_2 and A_3B_3 are the $(RRR)_p(RR)_p$ legs which have the wrench system $1-\hat{\$}^{\infty}$ corresponding to Fig. 2(a). With regard to the geometric analyses to come, the successive axes in the legs 1,2 and 3 will be referred to as e_{ik}, where $k = 1\ldots3$ indicates the leg number and $k = 1\ldots5$ is for the five revolute joints from the base to the platform. If the axes e_{ik} and e_{jk} form a plane, it would be referred by Π_{ijk}. The same nomenclature will be applied as well for the next mechanism candidates.

4.2.1. Geometric conditions between and within the legs

This mechanism candidate has one leg with the wrench system $1-\hat{\$}^{0}$ and two legs with the wrench system $1-\hat{\$}^{\infty}$. Hence the overconstraint is that of the supplementary wrench system $1-\hat{\$}^{\infty}$.

For the leg $(RRR)_p(RR)_i$, the axes of the revolute joints within the subchain $(RRR)_p$ are parallel to the vector $\mathbf{z_b}$ whereas those within the subchain $(RR)_i$ intersect in E.

For the legs $(RRR)_p(RR)_p$, the axes of the revolute joints within the subchain $(RRR)_p$ are parallel to vector $\mathbf{x_b}$ whereas those within the subchain $(RR)_p$ are parallel to each other. Between the two $(RRR)_p(RR)_p$ legs, the subchains $(RRR)_p$ have axes parallel to each other and similarly, the subchains $(RR)_p$ of the two legs have axes parallel to each other.

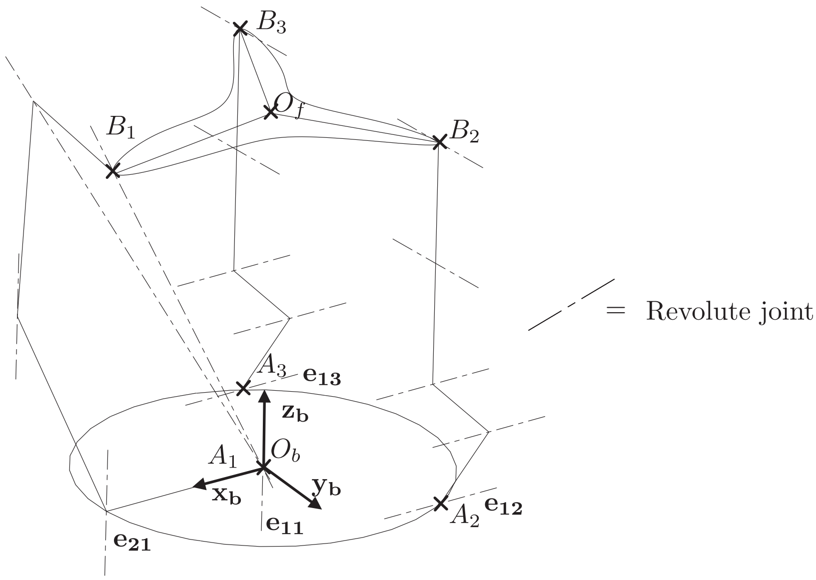


Fig. 11. 2T2R — Candidate II.

4.2.2. Mobility check

All the above conditions within and between legs are satisfied in all configurations, hence the parallel mechanism with a $1\text{–}(RRR)_p(RR)_i\text{–}2\text{–}(RRR)_p(RR)_p$ architecture has a full-cycle 2T2R mobility.

4.2.3. Choice of the inputs

As four inputs are needed to control this 2T2R parallel mechanism and this candidate has three legs, one of the legs needs to be assigned two inputs. One input is assigned to each $(RRR)_p(RR)_p$ leg and is located at the first revolute joint, namely e_{12}, e_{13} connected to the base whereas the other two inputs are assigned to the $(RRR)_p(RR)_i$ leg and are located at the first two joints starting from the base, namely e_{11}, e_{21}. This choice of the inputs needs to be validated to ascertain whether the applied actuation wrenches lock every DOF in a general configuration.

A formulation of unique form of screw based Jacobian for lower mobility parallel manipulators is given in [27] but the proposed form of the Jacobian matrix includes only the actuation wrenches and not the constraint wrenches. Hence, the alternate form of direct Jacobian matrix described in [28] is utilized, which includes the constraint wrenches. The validation of the actuated joints is conducted by analyzing the full direct Jacobian $\mathbf{J_{xII}}$ of the mechanism candidate II, which can be obtained by stacking the four actuation wrenches $\hat{\$}_{i,a}$ and the two constraint wrenches $\hat{\$}_{j,c}$ of the parallel mechanism. The four actuation wrenches can be expressed as $\hat{\$}_{i,a} \equiv [\mathrm{s_i} \quad \mathrm{s_i} \times \mathrm{r_i}]^T$ where, $\mathbf{s_i}$ denotes the direction of the actuation wrench and $\mathbf{r_i}$ is the position vector directed from a point of the wrench axis to the origin O_b. The two constraint wrenches $\hat{\$}_{j,c}$ produced by the mechanism have the form $\hat{\$}_{i,c} \equiv [\mathrm{z_b} \quad \mathrm{z_b} \times \mathrm{EO_b}]^T$ and $\hat{\$}_{2,c} \equiv [0 \quad \mathrm{m_1}]^T$ where $\mathbf{m_1}$ is the direction of the wrench system $1\text{–}\hat{\$}^{\infty}_{mp_1}$. Therefore the vectorial Jacobian of the parallel mechanism can be displayed as:

$$\mathbf{J_{xII}} = \begin{bmatrix} \mathbf{s_1} \times \mathbf{r_1} & \mathbf{s_1} \\ \mathbf{s_2} \times \mathbf{r_2} & \mathbf{s_2} \\ \mathbf{s_3} \times \mathbf{r_3} & \mathbf{s_3} \\ \mathbf{s_4} \times \mathbf{r_4} & \mathbf{s_4} \\ \mathbf{z_b} \times \mathbf{EO_b} & \mathbf{z_b} \\ \mathbf{m_1} & \mathbf{0} \end{bmatrix}.$$

It can be noticed that each row of this Jacobian is the transpose of the actuation and constraint wrenches written in axis coordinates. The choice of the selected inputs is invalid if the matrix $\mathbf{J_{xII}}$ is singular in all configurations. Actuated joints e_{11}, e_{21}, e_{12}, e_{13} are shown in Fig. 11.

With the notations defined before Section 4.2.1, a geometric interpretation of the actuation wrench system $4\text{–}\hat{\$}_a$ can be obtained. The actuation wrenches $\hat{\$}_{1,a}$ and $\hat{\$}_{2,a}$ are each defined by the intersection of the planes taken in the pairs (Π_{231}, Π_{451}) and (Π_{131}, Π_{451}). Thus, it can be concluded that $\hat{\$}_{1,a}$ and $\hat{\$}_{2,a}$ both lie on the plane Π_{451} and hence must intersect each other if not parallel to each other. Thus $\hat{\$}_{1,a}\text{–}\hat{\$}_{2,a}$ forms a planar pencil of lines. Similarly, actuation wrenches $\hat{\$}_{3,a}$ and $\hat{\$}_{4,a}$ are defined respectively by the intersection of the planes in the pairs (Π_{232}, Π_{452}) and (Π_{233}, Π_{453}). For simplicity it is assumed that each axes pair (e_{41}, e_{42}) and (e_{51}, e_{52}) contains coincident axes. This assumption makes Π_{452} and Π_{453}, coincident. Following this, it can be concluded that $\hat{\$}_{3,a}-\hat{\$}_{4,a}$ lies on the same plane and hence form a planar pencil of lines. The constraint wrench $\hat{\$}_{1,c}$ is a line passing through E and parallel to $\mathbf{z_b}$. The constraint wrench $\hat{\$}_{2,c}$ is a line orthogonal to the axes quintuple (e_{12}, e_{22}, e_{32}, e_{42}, e_{53}).

From the above discussion, the following three geometric conditions can be derived when the variety formed by $4-\hat{\$}_a$ and $2-\hat{\$}_c$ degenerates:

1. The wrenches $\hat{\$}_{1,a}$ and $\hat{\$}_{2,a}$ become coincident. This condition occurs, whenever planes (Π_{131}, Π_{231}) become coincident. This condition occurs at the boundary of the workspace.
2. The wrenches $\hat{\$}_{3,a}$ and $\hat{\$}_{4,a}$ become coincident. This condition occurs, whenever planes (Π_{232}, Π_{233}) become coincident. This condition occurs inside the workspace of the mechanism.

As the matrix $\mathbf{J_{xII}}$ is not singular for every possible configuration of the platform, the choice of the inputs is valid.

4.3. Architecture candidate III

For the synthesis of parallel mechanism with mobility corresponding to Fig. 2(b), the number of overconstraints n_{oc} is chosen to be zero. Hence, $n_{legs} = 2 + n_{oc}$ equals two. The first leg corresponding to the wrench system $1-\hat{\$}^{0}$ is the same as the one chosen for candidate II. For the second leg with $1-\hat{\$}^{\infty}$ Case-2, a closed loop kinematic chain with two points of attachment at the base, as shown in Fig. 10(b), is chosen. To comply with the nomenclature defined before Section 4.2.1, this closed loop kinematic chain can be considered as consisting of two legs connecting the platform at the prismatic joint. Thus, the architecture 1–$(RRR)_p(RR)_i$–2–$(RR)_i(RRR)_p$–P is obtained as a parallel mechanism candidate with a wrench system corresponding to Fig. 2(b). A wire model of this candidate III is shown in Fig. 12.

4.3.1. Geometric conditions between and within the legs

For the first leg $(RRR)_p(RR)_i$, the axes of the revolute joints within the subchain $(RRR)_p$ are parallel to vector $\mathbf{z_b}$, whereas the axes of the revolute joints within the subchain $(RR)_i$ intersect in E, which, in the reference configuration shown in Fig. 12, coincides with O_b.

Within the second 2–$(RR)_i(RRR)_p$–P leg, which consists of the closed kinematic chain $A_2A_3B_3B_2$, the axes of the revolute joints within the subchains $(RRR)_p$ are parallel to each other and to $\mathbf{y_f}$. The axes of the revolute joints within the subchains $(RRR)_i$ intersect at points P_2 and P_3, which lie on the axis (O_b, $\mathbf{x_b}$). The direction of the prismatic joint is parallel to the axes of the revolute joints in the subchains $(RRR)_p$ and to $\mathbf{y_f}$.

4.3.2. Mobility check

All the above conditions within and between legs are satisfied in all configurations, hence the architecture 1–$(RRR)_p(RR)_i$–2–$(RR)_i(RRR)_p$–P as 2T2R parallel mechanism has a full-cycle mobility.

4.3.3. Choice of the inputs

As with candidate II, four inputs are needed to control this 2T2R parallel mechanism. This candidate has two legs, with two inputs assigned to each. The $(RRR)_p(RR)_i$ leg is assigned two inputs to the first two revolute joints from the base. The chain

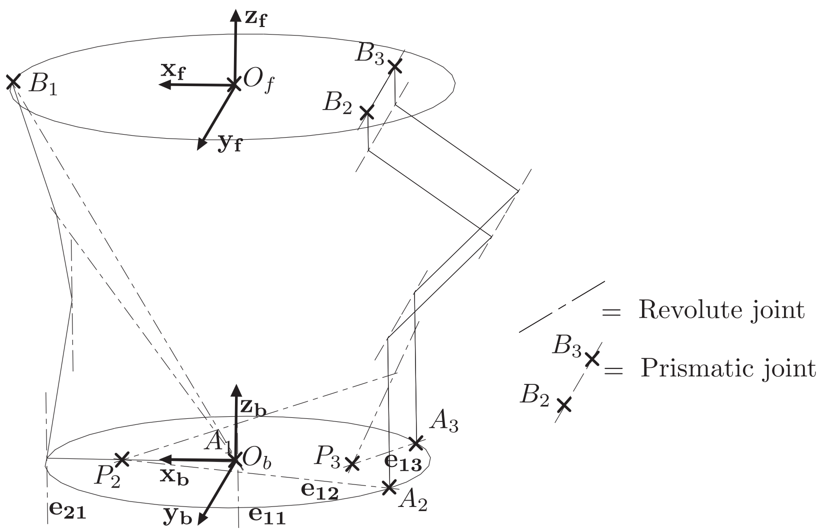


Fig. 12. 2T2R — Candidate III.

2–$(RR)_i(RRR)_p$–P is assigned two inputs to the two revolute joints attached to the base. The actuated joints e_{11}, e_{21}, e_{12} and e_{13} are shown in Fig. 12. The full direct Jacobian matrix $\mathbf{J}_{\mathbf{xIII}}$ for this candidate is displayed below:

$$\mathbf{J}_{\mathbf{xIII}} = \begin{bmatrix} \mathbf{s_1} \times \mathbf{r_1} & \mathbf{s_1} \\ \mathbf{s_2} \times \mathbf{r_2} & \mathbf{s_2} \\ \mathbf{m_1} & \mathbf{0} \\ \mathbf{m_2} & \mathbf{0} \\ \mathbf{z_b} \times \mathbf{EO_b} & \mathbf{z_b} \\ \mathbf{z_f} & \mathbf{0} \end{bmatrix}.$$

As before, the first four rows of $\mathbf{J}_{\mathbf{xIII}}$ correspond to the four actuation wrenches $\hat{\$}_{1,a}$ with $i = 1\ldots4$ and the last two rows correspond to the two constraint wrenches $\hat{\$}_{1,c}$ with $1\ldots2$.

Let the axis of the prismatic joint be denoted by e_6. In addition, let the planes orthogonal to the axes e_{22} and e_{23} be denoted by π_1 and π_2 respectively and the planes orthogonal to the axis sets $\{e_{32}.\ e_{42},\ e_{52}\}$ and $\{e_{33}.\ e_{43},\ e_{53}\}$ be denoted by π_3 and π_4 respectively. The planes π_3 and π_4 are perpendicular to $\mathbf{y_f}$ and hence also parallel to each other.

With this notation, a geometric interpretation of the actuation wrench system $4-\hat{\$}_a$ can be obtained. The description of the actuation wrenches $\hat{\$}_{i,a}$ and $\hat{\$}_{2,a}$ is the same as for candidate II, as one leg is common to all the candidates. The axes of the actuation wrenches $\hat{\$}_{3,a}$ and $\hat{\$}_{4,a}$ are the intersection of the plane couples (π_1, π_3) and (π_2, π_4), respectively. As the sets of axes $\{e_{32}, e_{42}, e_{52}\}$ and $\{e_{33}, e_{43}, e_{53}\}$ are parallel to each other, the planes π_3 and π_4 are also parallel to each other.

The axis of the constraint $\hat{\$}_{1,c}$ is a line passing through E and parallel to $\mathbf{z_b}$, whereas the axis of $\hat{\$}_{2,c}$ is a line parallel to $\mathbf{z_f}$ and hence also parallel to π_3 and π_4. Finally, the axes of $\hat{\$}_{3,a}$, $\hat{\$}_{4,a}$ and $\hat{\$}_{2,c}$ are all parallel to the planes π_3 and π_4, hence are linearly dependent in all configurations. Thus, the matrix $\mathbf{J}_{\mathbf{xIII}}$ is singular in all configurations. Hence, the choice of inputs for the actuated joints is not valid for this architecture candidate.

4.4. Architecture candidate IV

As the choice of the inputs for candidate III could not be validated, a second variant architecture candidate corresponding to the constraint wrench 2(b) is now proposed. This architecture is derived from candidate III in which the two subchains $(RR)_i(RRR)_p$ in the loop $A_2A_3B_3B_2$ have been replaced by two subchains $(RRR)_i(RR)_p$ of higher complexity. These two subchains $(RR)_i(RRR)_p$ and $(RRR)_i(RR)_p$ are interchangeable as mentioned at the end of Section 3.6, since both have the same $1-\hat{\$}^0$ wrench system. The number of overconstraints n_{oc} is zero which is the same as for candidate III. A similar nomenclature applies as for candidate III and a wire model of this for candidate IV is shown in Fig. 13.

4.4.1. Geometric conditions between and within the legs

The geometric conditions for the leg $(RRR)_p(RR)_i$ are similar to those of candidate III. Within the second 2–$(RRR)i(RR)_p$–P leg which consists of the closed kinematic chain $A_2A_3B_3B_2$, the axes of the revolute joints within the subchains $(RR)_p$ are parallel to $\mathbf{y_f}$, whereas the axes of the revolute joints within the subchains $(RRR)_i$ intersect at P_2 and P_3. These latter points lie on the axis $(O_b, \mathbf{x_b})$. The direction of the prismatic joint is parallel to the axis of the revolute joints in the subchains $(RR)_p$ and to the vector $\mathbf{y_f}$.

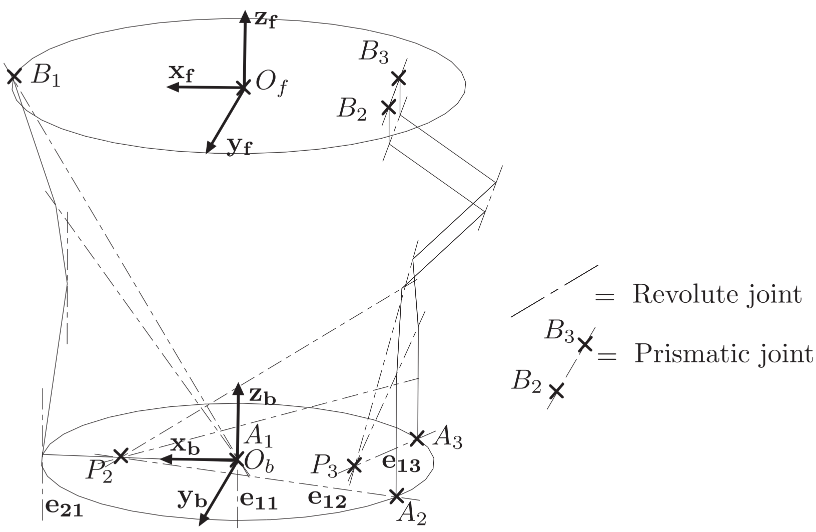


Fig. 13. 2T2R — Candidate IV.

4.4.2. Mobility check

All the above conditions within and between legs are satisfied in all configurations. Hence, the architecture candidate $1–(RRR)_p(RR)_i)–2–(RRR)_i(RR)_p–P$ as a 2T2R parallel mechanism has a full-cycle mobility.

4.4.3. Choice of the inputs

As with previous candidates, four inputs are needed to control this 2T2R parallel mechanism. The inputs assignment is similar to that of candidate III leading to the four actuated joints e_{11}, e_{21}, e_{12} and e_{13}, as displayed in Fig. 13. The full direct Jacobian matrix $\mathbf{J_{xIV}}$ for this candidate is given below:

$$\mathbf{J_{xIV}} = \begin{bmatrix} \mathbf{s_1} \times \mathbf{r_1} & \mathbf{s_1} \\ \mathbf{s_2} \times \mathbf{r_2} & \mathbf{s_2} \\ \mathbf{s_3} \times \mathbf{r_3} & \mathbf{s_3} \\ \mathbf{s_4} \times \mathbf{r_4} & \mathbf{s_4} \\ \mathbf{z_b} \times \mathbf{EO_b} & \mathbf{z_b} \\ \mathbf{z_f} & \mathbf{0} \end{bmatrix}.$$

where the actuation wrenches $\hat{\$}_{1,a}$ and $\hat{\$}_{2,a}$ are defined similarly to candidate II. However, the derivation of the actuation wrenches $\hat{\$}_{3,a}$ and $\hat{\$}_{4,a}$ is not straightforward as in previous cases.

Let us define two intermediate wrenches $\hat{\$}^*_{3,a}$ and $\hat{\$}^*_{4,a}$ whose axes are respectively the intersection of the plane couples (Π_{232}, Π_{452}) and (Π_{233}, Π_{453}). As can be readily seen, the wrenches $\hat{\$}^*_{3,a}$ and $\hat{\$}^*_{4,a}$ are not reciprocal to the twist $\hat{\$}^\infty_6$ associated with the prismatic joint which has a direction parallel to $\mathbf{y_f}$.

Below, the expressions for $\hat{\$}_{3,a}$ and $\hat{\$}_{4,a}$ are given without any developed proof but it is easy to verify that these two actuation wrenches are reciprocal to $\hat{\$}^\infty_6$:

$$\hat{\$}_{3,a} = \hat{\$}^*_{3,a} - \left(\hat{\$}^*_{3,a} \cdot \hat{\$}^\infty_6\right) \hat{\$}^*_{1,c}$$
$$\hat{\$}_{4,a} = \hat{\$}^*_{4,a} - \left(\hat{\$}^*_{4,a} \cdot \hat{\$}^\infty_6\right) \hat{\$}^*_{2,c}$$

where $\hat{\$}^*_{1,c}$ and $\hat{\$}_{2,c}$ are the lines passing through P_2 and P_3 respectively and parallel to the vector $\mathbf{y_f}$. The axes of $\hat{\$}^*_{3,a}$ and $\hat{\$}^*_{4,a}$ are both orthogonal to $\mathbf{y_f}$ and parallel to the planes Π_{452} and Π_{453}, respectively. From the above discussion, the following geometric conditions leading to the degeneracy of the variety formed by the wrench systems $4-\hat{\$}_a$ and $2-\hat{\$}_c$ can be formulated:

1. The planes (Π_{131}, Π_{231}) become coincident. In such a case, wrenches $\hat{\$}_{1,a}$ and $\hat{\$}_{2,a}$ become coincident. This condition is the same as that for candidate II.
2. The planes Π_{452} and Π_{453} are coincident and parallel to the plane formed by $\mathbf{x_f}$ and $\mathbf{y_f}$. In this case, $\hat{\$}_{3,a}$ and $\hat{\$}_{4,a}$ constitute a 2-system which includes a moment with a direction parallel to $\mathbf{z_f}$. Hence, the variety formed by $\hat{\$}_{3,a}$, $\hat{\$}_{4,a}$ and $\hat{\$}_{2,c}$ degenerates to a 2-system.

As the matrix $\mathbf{J_{xIV}}$ is not singular for every possible configuration of the platform, the choice of the inputs is valid for $1–(RRR)_p(RR)_i–2–(RRR)_i(RR)_p–P$ as 2T2R parallel mechanism.

4.5. Summary

Candidate	I	II	III	IV
Motion Pattern	mp_1	mp_1	mp_2	mp_2
Full-cycle mobility	No	Yes	Yes	Yes
Choice of actuation valid	–	Yes	No	Yes

From the above discussion on the four architecture candidates, it could be inferred that candidates II and IV are feasible candidates as they satisfy the essential criteria of full-cycle mobility and validity of the selection of the actuated joints.

5. Conclusion

A generic task-based procedure for the synthesis of parallel mechanisms is presented and illustrated with the task of needle manipulation motivated by a medical application. While stressing on the importance of the architecture kinematic complexity, a systematic methodology based on screw theory was demonstrated for the inclusion of preference rules at an early stage of the design process. Two different novel motion patterns, corresponding to 2T2R mobility, were derived for the first time and consequently, two entirely different parallel mechanisms having the same mobility were synthesized. These mechanisms have simpler architecture in terms of the geometric complexity but following the same procedure, it is possible to derive more complex variants that would fit the same task. Since the validity of the selection of actuated joints and the conditions of full-cycle mobility have been verified, designers can proceed with more confidence for the next stages of dimensional synthesis as well as embodiment design.

This study also reinforces the understanding that the concept of mobility is not sufficient for mechanism synthesis purposes and should be complemented with motion patterns that convey more accurately the motions and constraints expected for the desired mechanism. During the synthesis process, application of the principle of kinematic inversion was demonstrated for the synthesis of parallel mechanisms in special cases where the motion pattern is specified with respect to the end-effect or reference frame. This method could be applied to other syntheses where motion or constraint description is given with respect to a moving reference frame instead of a base frame.

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